Minimum-Latency Broadcast Scheduling for Cognitive Radio Networks

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Abstract—Cognitive Radio Networks (CRNs) introduce a new communication paradigm which enable unlicensed users to opportunistically access spectrum bands assigned to licensed users. Interestingly, the broadcast problem, which is one of the most fundamental operations in CRNs, has not been well studied. Existing works for the broadcast issue in CRNs are either heuristic solutions without performance guarantee or with performance far from the optimal solution. In this paper, we study the Minimum-Latency Broadcast Scheduling (MLBS) issue for CRNs. Our contributions are threefold. Firstly, we propose a Mixed Broadcasting Scheduling (MBS) algorithm under the Unit Disk Graph (UDG) model, denoted by MBS-UDG. MBS-UDG finishes a broadcast task by employing mixed unicast and broadcast communication modes in two phases. We show that the latency performance of MBS-UDG is $O(\hbar + \Delta_T)$ when $\Delta_T \leq 1/p$, or $O(\hbar + \log_{1-p} \frac{1}{p\Delta_T})$ when $\Delta_T > 1/p$, where $\hbar$ and Δ_T are the height and the maximum number of leaf nodes connected by a SU of the broadcasting tree, respectively, and p is the spectrum opportunity for a secondary communication. Furthermore, the redundancy performance of MBS-UDG is analyzed. Secondly, we extend MBS-UDG to a more general MBS algorithm under the protocol interference model and analyze its latency and redundancy performance. Finally, simulations are conducted to validate MBS, which indicate that MBS significantly improves existing algorithms with respect to both latency and redundancy.

Index Terms—Cognitive Radio Networks (CRNs); broadcast; scheduling; minimum-latency; redundancy; Unit Disk Graph (UDG) model; protocol interference model.

I. INTRODUCTION

Wireless spectrum is one of the most precious resources. With the rapid growth of the number of wireless devices, communications over the unlicensed spectrum bands become very crowded. On the other hand, the utilization of the spectrum assigned to licensed users varies from 15% to 85% according to the report from the Federal Communications Commission (FCC) [1], which is very inefficient. To alleviate the interference and collisions on the unlicensed spectrum, as well as to improve the efficiency of the licensed spectrum, researchers propose a new communication paradigm recently, named Cognitive Radio Networks (CRNs), which enable unlicensed users to sense and learn the communication environment with an equipped cognitive radio, and opportunistically access the licensed spectrum without causing any unacceptable interference to the activities of licensed users [19][20][21].

Under the CRN model, a *secondary network* consisting of Secondary Users (SUs) (unlicensed users) coexists with a *primary network* consisting of Primary Users (PUs) (licensed users). They share the same time, space, and spectrum. For a SU, it senses and learns its local wireless environment before initializing a data transmission. If there is a spectrum opportunity, this SU can conduct the data transmission without hurting any activity of the primary network.

Ever since the CRN communication model is introduced, many efforts have been spent on various issues in CRNs. In this paper, we study the Minimum-Latency Broadcast Scheduling (MLBS) problem for CRNs. Broadcast is one of the most fundamental operations in wireless networks, as well as in CRNs, which aims to deliver a message from a source to all the other nodes. In multi-hop CRNs, the *broadcast latency* is defined as the time consumption by which all the nodes in the network have received the broadcast message from the source directly or via a multi-hop manner. Since time is a major concern for many CRN operations, we study the MLBS problem where we try to seek a broadcast scheduling strategy with the minimum broadcast latency.

The MLBS problem is NP-hard in traditional wireless networks even under the simple Unit Disk Graph (UDG) model [3]. Therefore, many approximate algorithms are proposed for MLBS in traditional wireless networks [3]-[7]. However, it is not trivial to adopt these algorithms to CRNs due to the following reasons. CRN is a new communication paradigm, which consists of two networks with different priorities. The activity of the primary network introduces many constraints on the spectrum accessibility of SUs. Unlike traditional wireless networks, where nodes can access the spectrum freely, SUs have to sense and learn its local wireless environment. Only when there are spectrum opportunities for SUs, i.e. the activity of SUs does not cause any unacceptable interference to the primary network, they can access the spectrum. Furthermore, the activities of PUs also induce more interference to SUs besides the interference produced in the secondary network. Therefore, when design broadcast scheduling algorithms for CRNs, elegant techniques are required to consider spectrum opportunities and wireless interference together.

Recently, a few works [9][10][11][12] considered the broadcast scheduling issue for CRNs. In [9], Song and Xie proposed a distributed broadcast protocol without employing a common control channel. The proposed protocol is also unaware of global network topology or time synchronization information. Nevertheless, explicit broadcast latency bound is not gave

in that work. In [10], Kondareddy and Agrawal studied the selective broadcasting problem in CRNs, in which a message is transmitted over a pre-selected set of channels. They proposed a simple heuristic solution and validated their method by simulations. In [11], a broadcasting algorithm is designed for CRNs to maximize the message reachability and reduce the data redundancy and propagation latency. However, analysis for the algorithm is not provided neither, and only simulations are conducted to examine the algorithm. For the MLBS problem, Arachchige *et al.* [12] recently proposed two heuristics with time complexities of $O(RMN^3 \log N + LN^3 \log N)$ and $O(LMN^3 \log N)$ respectively, where R is the network radius, M is the number of available channels in a CRN, N is the number of SUs, and L is the number of time slots in one-time schedule. However, the time performances of these two heuristics are far from the optimum solution ($O(R)$). In this work, we significantly improve these two results.

Considering that existing works for the broadcast problem in CRNs are either heuristic solutions without performance guarantee or with performance far from the optimal solution, we first propose a Mixed Broadcast Scheduling (MBS) algorithm under the *Unit Disk Graph* (UDG) model, which tries to finish the broadcast scheduling by wisely exploiting unicast and broadcast collaboratively. Subsequently, we theoretically analyze the proposed algorithm and obtain its induced broadcast latency and redundancy, which remedies the gap of existing works. To be more general, we further extend MBS to the CRNs under the *Protocol Interference Model* (PrIM), and also analyze its latency and redundancy performance. In summary, the main contributions of this paper are as follows.

1) For the MLBS problem in CRNs, we propose a Mixed Broadcast Scheduling (MBS) algorithm under the UDG model, denoted by MBS-UDG. MBS-UDG consists of two phases. In the first phase, the broadcast packet is delivered to all the backbone SUs of the broadcasting tree. Subsequently, all the dominator SUs of the broadcasting tree are partitioned into disjoint and interference-free subsets by hexagonal tessellation and coloring. Then, in Phase II, these subsets are scheduled concurrently and repeatedly to finish the broadcast task by employing mixed unicast and broadcast communication modes. By theoretical analysis, we show that the broadcast latency of MBS-UDS is $(367(\hbar - 1) + 12\Delta_T)/p$ time slots if $\Delta_T \leq 1/p$, and $367(\hbar - 1)/p + 12/p^2 + 12e^{\pi\lambda r^2} \log_{1-p} \frac{1}{p\Delta_T}$ time slots if $\Delta_T > 1/p$, where $\hbar$ is the height of the broadcasting tree (similar as the network radius), Δ_T is the maximum number of dominatee children (leaf children) connected by a dominator in the broadcasting tree, λ is the transmission intensity of PUs, r (resp., R) is the transmission range of SUs (resp., PUs), and $p = \exp(-\pi\lambda(r^2 + R^2))$ is the spectrum opportunity for a secondary communication. Evidently, this result is a significantly improvement over [12]. We also analyze the broadcast redundancy of MBS-UDG, which is defined as the maximum transmission times

of the broadcast packet by any SU in the broadcast scheduling. The broadcast redundancy of MBS-UDG is upper bounded by $12 + \Delta_T$ when $\Delta_T \leq 1/p$, and $12 + 1/p + \log_{1-p} \frac{1}{p\Delta_T}$ when $\Delta_T > 1/p$.

2) We extend MBS-UDG to CRNs under the PrIM, denoted by MBS-PrIM. By similar techniques employed in MBS-UDG, we also show the broadcast latency and redundancy of MBS-PrIM.

3) (iii) Finally, extensive simulations are conducted to validate the performance of MBS. Simulation results indicate that MBS significantly improves both the broadcast latency and the broadcast redundancy over existing algorithms. Specifically, MBS induces over 290% and 840% less latency and redundancy than existing algorithms, respectively.

The rest of this paper is organized as follows. In Section II, we summarize existing works.In Section III, the network model, interference model and problem definition are presented. The broadcasting tree is constructed in Section IV. In Section V, the Mixed Broadcast Scheduling (MBS) algorithm under the UDG model is designed and analyzed. To be more general, we also extend MBS to CRNs under the protocol interference model.In Section VI, extensive simulations are conducted to examine the performance of MBS. Finally, we conclude this paper in Section VII.

II. RELATED WORK

Broadcast scheduling, especially MLBS, is a fundamental problem in wireless networks. In this section, we will summarize some existing works on MLBS for traditional wireless networks. Subsequently, the works on the broadcasting issue for CRNs will also be discussed.

A. Broadcast Scheduling in Traditional Wireless Networks

In [3],Gandhi *et al.* first studied the MLBS problem under the UDG model for ad hoc networks. They proved the NP-hardness of the MLBS problem and subsequently proposed an approximation broadcast scheduling algorithm. However, the approximation guarantee of their algorithm is greater than 400. Later on, Huang *et al.* in [4] improved the algorithm in [3].They designed two algorithms with approximation guarantee of 51 and 24 respectively, and one more efficient broadcast scheduling algorithm of latency $R + O(\sqrt{R} \log^{1.5} R)$, where R is the network radius. Since R is a trivial lower latency bound of any broadcast scheduling algorithm and $O(\sqrt{R} \log^{1.5} R) < O(R)$ when $R \to \infty$, the third algorithm in [4] is nearly optimal with respect to order. Huang *et al.* further improved the ratio bounds of their algorithms in [5], in which three approximation algorithms are designed with latency upper bounded by $24R - 23$, $16R - 15$, and $R + O(\log R)$, respectively.

In [8], Chen *et al.* investigated the MLBS problem for traditional wireless networks under the protocol interference model. They designed a broadcast scheduling algorithm of approximation ratio 26 under the UDG model and $2\pi\alpha^2$ under the protocol interference model, where α is the ratio

between the interference range and the transmission range of a node. Similarly, Mahjourian *et al.* [6] also studied the MLBS problem for wireless ad hoc networks under the protocol interference model. They designed a broadcast scheduling algorithm of latency upper bounded by $O((\max(\alpha, \beta))^2)R$, where β is the ratio between the carrier sensing range and the transmission range of a node.

B. Broadcast Scheduling in CRNs

As a new promising communication paradigm, CRNs attract lots of attention recently. However, for the broadcast scheduling problem, as one of the most fundamental issues in CRNs, only a few existing works consider it [10]-[12]. In [10], Kondareddy and Agrawal studied the selective broadcasting problem in multi-hop CRNs, where the broadcast message is transmitted over a pre-selected set of channels. By introducing the concepts of neighbor graphs and minimal neighbor graphs, a heuristic selective broadcasting algorithm is designed. Furthermore, the authors also took simulations to verify the proposed algorithm. In [11], a broadcasting algorithm is proposed with objectives to maximize the reachability of the broadcast message, and reduce the redundancy and propagation latency. However, no quantitative analysis is provided except for simulations. Recently, Arachchige *et al.* [12] studied the MLBS problem for CRNs under a simple network interference model. They first formulated the MLBS problem as an Integer Linear Programming (ILP) problems and then proposed two heuristics. However, the time complexities of their heuristics are $O(RMN^3 \log N + LN^3 \log N)$ and $O(LMN^3 \log N)$ respectively, which are far from the optimum solution.

III. SYSTEM MODEL AND PROBLEM DEFINITION

In this section, we give the network model and interference model. We also formally define the studied minimum-latency broadcast scheduling problem in CRNs.

A. Network Model

We consider a secondary network coexisted with a primary network. They share the same time, space, and spectrum. Particularly, we consider the general single-spectrum model, which is reasonable for protocol design and analysis in CRNs as indicated by many existing works [17][18].

Primary Network: The primary network consists of N Poisson distributed Primary Users (PUs) denoted by set $V_p = \{S_1, S_2, \cdots, S_N\}$. The transmission radius and interference radius of PUs are defined as R and R_I, respectively. The network time is assumed to be slotted with each time slot of length τ, and a PU can transmit one data packet during a time slot. At the very beginning of each time slot, each PU decides to transmit some data or keeps silent in that time slot according to the working protocol of the primary network (the case that a PU to be a receiver can be treated as silent since it will not cause any interference to other PUs or SUs). Furthermore, during each time slot, the primary transmitters are distributed according to a two-dimensional Poisson point process X_T with density λ. Therefore, according to the *Displacement Theorem*

[13], the distribution of primary receivers during each time slot is correlated with X_T, and forms another two-dimensional Poisson point process X_R also with density λ.

Secondary Network: The considering secondary network is coexisted with the primary network, which consists of one randomly distributed *broadcasting source* Secondary User (SU) denoted by s_0, and n randomly distributed SUs denoted by $s_1, s_2, \cdots, s_n$. The transmission radius and interference radius of SUs are represented by r and r_I, respectively. For a pair of SUs s_i and s_j, there is a link between them *iff* the Euclidean distance $D(s_i, s_j)$ between them satisfies $D(s_i, s_j) \leq r$. Therefore, the secondary network can be modeled by a graph $G = (V_s, E_s)$, where $V_s = \{s_0, s_1, s_2, \cdots, s_n\}$ is the node set and E_s is the set of all the possible links formed by SUs in V_s. To make the broadcast scheduling problem meaningful, we assume G is connected.

Note that in a CRN, even there is a link between SUs s_i and s_j, this does not imply s_i can transmit a data packet to s_j during a time slot because of lacking of spectrum opportunities. Therefore, to successfully transmit a data packet to s_j from s_i, three more conditions need to be satisfied: (i) there are no PUs within the interference range of s_i and receiving some data; (ii) there are no PUs such that s_j is within the interference range of them and they are transmitting some data; (iii) the communication between s_i and s_j is interference-free. Consequently, when we design broadcast scheduling algorithms, we have to consider interference avoiding and spectrum opportunities together.

B. Interference Model

In this paper, we consider two frequently exploited interference models: the UDG model and PrIM.

UDG Model: The UDG model is a widely used interference model in existing works [3]-[7], which is simple and very convenient in performance analysis. In the UDG model, the interference range is equal to the transmission range, which implies $R_I = R$ and $r_I = r$ in our network model.

PrIM: The PrIM is a general version of the UDG model, in which the interference range of a node is assumed to be greater than or equal to the transmission range of that node. Hence, in our network model, we assume $R_I = \beta \cdot R$ and $r_I = \beta \cdot r$, where $\beta \geq 1$ is a constant value.

C. Problem Definition

Based on the defined network model and interference model, we can formally define the MLBS problem for CRNs as follows. Given a secondary network represented by graph $G = \{V_s = \{s_0, s_1, s_2, \cdots, s_n\}, E_s\}$, where s_0 is the broadcasting source and s_0 holds a data packet which needs to send to all the other SUs in the network. Then, a broadcast scheduling of latency ℓ is a sequence of subsets $\langle V_0, V_1, V_2, \cdots, V_\ell \rangle$ satisfying that

1) $V_0 = \{s_0\}$ and $\forall 1 \leq i \leq \ell, V_i \subseteq V_s$;
2) During time slot $1 \leq i \leq \ell$, all the SUs in V_i can receive the broadcast data packet from some SUs in $\bigcup_{j=0}^{i-1} V_j$ successfully;
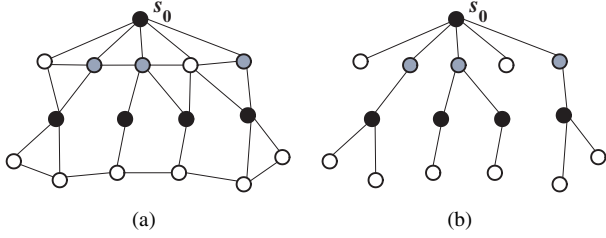
Fig. 1. Construction of the CDS-based Broadcasting Tree.

3) $\bigcup_{i=0}^{\ell} = V_s$, which implies all the SUs have a copy of the broadcast data packet at the end of the scheduling.

The MLBS problem is to seek a broadcast scheduling strategy of minimum ℓ. Its *broadcast latency* is defined as ℓ, and its *broadcast redundancy* is defined as $\max\{d_i | s_i \ (0 \leq i \leq n)$ transmits d_i times of the broadcast data packet during the entire MLBS process$\}$. Evidently, when design a MLBS algorithm, besides to minimize the latency ℓ, we also want to have low broadcast redundancy, which implies less energy consumption and spectrum requirement. The MLBS problem is NP-hard even in conventional wireless networks under the UDG model. Therefore, we design an approximation algorithm with latency and redundancy analysis under the UDG model and the PrIM for CRNs.

IV. BROADCASTING TREE AND COLORING

In this section, we construct a *Connected Dominating Set* (CDS) based broadcasting tree, denoted by T, which serves as the scheduling infrastructure. Subsequently, we study the tessellation and coloring of a plane, which is useful in the designed interference-free scheduling algorithm.

A. CDS-based Broadcasting Tree

For the secondary network represented by graph $G = (V_s, E_s)$, a *Dominating Set* (DS) of G is a subset $\mathcal{D}$ of V_s such that $\forall s_i \in V_s$, either $s_i \in \mathcal{D}$ or s_i is adjacent to some SU in $\mathcal{D}$. If the induced subgraph $G[\mathcal{D}]$ of G on $\mathcal{D}$ is connected, $\mathcal{D}$ is a *Connected Dominating Set* (CDS) of G. A *Maximal Independent Set* (MIS) $\mathcal{M}$ of G is a subset of V_s such that (i) $\forall s_i \in V_s$, either $s_i \in \mathcal{M}$ or $\exists s_j \in \mathcal{M}$, $(s_i, s_j) \in E_s$; and (ii) $\forall s_i, s_j \in \mathcal{M}$, $(s_i, s_j) \notin E_s$. Clearly, an MIS is also a DS.

Employing a CDS as the infrastructure of a wireless network is widely adopted in existing literatures. In this paper, we construct a CDS-based broadcasting tree T according to the following three steps. First, make a Breadth First Searching (BFS) on G starting from s_0, and identify an MIS, denoted by $\mathcal{D}$ of G. As shown in Fig.1 (a), the set of black nodes is an MIS of that network. Evidently, $\mathcal{D}$ is also a DS. Usually, the black nodes in $\mathcal{D}$ are called *dominators*. Second, find a minimal set, denoted by $\mathcal{C}$, of nodes to connect the dominators in $\mathcal{D}$ to make $\mathcal{D} \cup \mathcal{C}$ a CDS of G. As shown in Fig.1 (a), $\mathcal{C}$ consists of blue nodes, which are also named *connectors*. The black nodes and blue nodes together form a CDS in Fig.1 (a). Finally, for every remaining white node in $V_s \setminus (\mathcal{D} \cup \mathcal{C})$, also called a *dominatee*, assign its neighboring dominator with the

shortest distance to s_0 as its parent node. For every connector (resp., dominator except for s_0) in $\mathcal{C}$ (resp., $\mathcal{D}$), assign its neighboring dominator (resp., connector) with the shortest distance to s_0 as its parent. Then, a CDS-based broadcasting tree T is constructed, e.g. Fig.1 (b) shows the broadcasting tree constructed for the secondary network in Fig.1 (a).

For $s_i \in V_s$ in T, define $p(s_i)$ $(s_i \neq s_0)$ as its parent node and $c(s_i) = \{s_j | p(s_j) = s_i\}$ as its children set. Let $\Delta(s_i) = |c(s_i) \cap (V_s \setminus (\mathcal{D} \cup \mathcal{C}))|$ be the number of dominatee children s_i has in T. Clearly, if s_i is a connector or dominatee, $\Delta(s_i) = 0$. We also define $\Delta_T = \max\{\Delta(s_i) | s_i \in V_s\}$, which indicates the maximum number of dominatee children that a dominator may have. The *height* of s_0 in T is defined to be $h(s_0) = 0$ and for the other node $s_i \neq s_0$, its *height* in T is defined as $h(s_i) = h(p(s_i)) + 1$. Furthermore, the *height* of T is defined as $\hbar = \max\{h(s_i) | s_i \in V_s\}$.

Now we introduce some important properties of T, which will be used in our algorithm, in Lemma 1. For, Lemma 1, Lemma 1.1 has been proven in [14], Lemma 1.2 can be easily proven based on the *Wegner Theorem* [15], and Lemma 1.3 is a result in [14].

Lemma 1: 1) $\forall s_i \in \mathcal{C}$, s_i is adjacent to at most 5 dominators in $\mathcal{D}$, of which one is its parent node. $\forall s_i \in \mathcal{D}$, $|c(s_i) \cap \mathcal{C}| \leq 11$ if $s_i \neq s_0$ and $|c(s_0) \cap \mathcal{C}| \leq 12$, i.e. s_i has at most 11 connector children if s_i is a non-root (not s_0) dominator and s_0 has at most 12 connector children; 2) There are at most 21 (resp., 42) dominators within a disk of radius $2r$ (resp., $3r$); 3) Generally, suppose the number of dominators within a disk of radius θr is ϕ_θ. Then $\phi_\theta \leq \frac{2\pi}{\sqrt{3}}\theta^2 + \pi\theta + 1$.

B. Tessellation and Coloring

A *tessellation* of a plane is to cover this plane with a pattern of flat shapes so that there are no overlaps or gaps. A *regular tessellation* is a pattern made by repeating a *regular polygon*. In existing literatures [7], *hexagonal tessellation* is a frequently used regular tessellation method to partition a network into hexagons. Similarly, for a CRN, we can use half-open half-closed hexagons of radius $r/2$ as shown in Fig.2 (a) to partition the network into hexagons as shown in Fig.2 (c). For a hexagonal tessellation, many coloring methods are proposed to color it. For instance, we give a 12-coloring unit of hexagons in Fig.2 (b), and its corresponding coloring of the tessellation is shown in Fig.2 (c). There are also many other 12-coloring methods for a hexagonal tessellation [7][16]. From Fig.2 (b) and (c), we can see that the hexagons of the same color are separated by at least the distance of $4 \cdot \frac{r}{2} = 2r$. Generally, we have a conclusion as shown in Lemma 2 [7][16].

Lemma 2: In a $3k^2$-coloring of a hexagonal tessellation, hexagons with the same color are separated by distance of at least $(3k - 2) \cdot \gamma$, where γ is the radius of the hexagons.

Now, we tessellate the considering CRN with half-open half closed hexagons of radius $\gamma = \frac{r}{2}$. Consequently, each hexagon contains at most one dominator of $\mathcal{D}$ in the broadcasting tree T. We further make a $3k^2$-coloring of the hexagonal tessellation, and assign each dominator the same color as the hexagon it belongs to. Then, $\mathcal{D}$ can be partitioned into $3k^2$
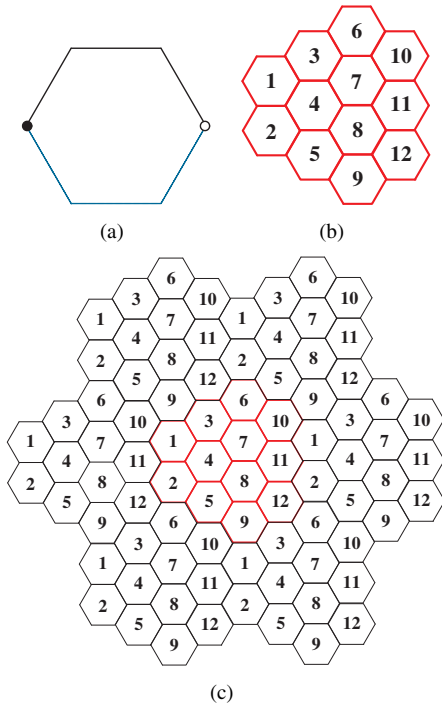
Fig. 2. (a) A half-open (the below half and the right-most vertex) half-closed (the above half and the left-most vertex) hexagon; (b) a basic 12-coloring unit of hexagons where each color is represented by an positive integer number; and (c) a hexagonal tessellation and its 12-coloring.

disjoint subsets $\mathcal{D}_1, \mathcal{D}_2, \cdots, \mathcal{D}_{3k^2}$ with $\mathcal{D}_i = \{s_i | s_i \in \mathcal{D}$, and s_i is assigned color $i\}$ for $1 \leq i \leq 3k^2$. Based on Lemma 2 and considering that all the hexagons are half-open half closed, we can conclude that $\forall s_u, s_v \in \mathcal{D}_i, D(s_u, s_v) > (3k-2) \cdot \gamma = \frac{(3k-2)r}{2}$.

V. BROADCAST SCHEDULING UNDER THE UDG MODEL

In this section, we first consider the MLBS problem under the UDG model and propose a broadcast scheduling algorithm. We then analyze its latency and redundancy. Finally, we extend the proposed algorithm to the more general PrIM.

A. MLBS under the UDG Model

Before giving the broadcast scheduling algorithm, we make some assumption on the size of the broadcast data packet first. In Section III, we assume the network is slotted with unit τ, and a PU can transmit one data packet during a time slot. For the broadcast data packet in the secondary network, we assume its size is smaller than the data packet transmitted in the primary network[1]. Therefore, we can partition a time slot into two parts τ_s, which is the *sensing window* for SUs to sense local wireless environments, and τ_d, which is the *data transmission window* and longer enough to transmit the broadcast data packet. Then, in our designed broadcast scheduling algorithm, for a SU s_i with data for transmission, it can randomly choose a *sensing point* τ_p in the sensing windowto sense

[1]Note that, if the broadcast data packet has a large size, we can break it into several smaller packets to accommodate our algorithm.

the local communication environment. If there is a spectrum opportunity, s_i can initialize a data transmission immediately after the sensing point according to the scheduling protocol. Furthermore, if multiple SUs try to transmit data during the same time slot, we assume no two SUs will chose the exactly same sensing point in the sensing window[2], which implies no two SUs will start to transmit data at the exactly same time point.

Algorithm 1: MBS-UDG

input : CDS-based broadcasting tree T
output: a broadcast scheduling plan

1 $\mathcal{B}_0 \leftarrow (\mathcal{C} \cup \mathcal{D}) \setminus \{s_0\}$;
2 $\mathcal{B}_1 \leftarrow \{s_0\}$;
3 **while** $\mathcal{B}_0 \neq \emptyset$ **do**
4   **for** *every* $s_i \in \mathcal{B}_1$ **do**
5   **if** $c(s_i) \cap \mathcal{B}_0 \neq \emptyset$ **then**
6   **for** *every* $s_j \in c(s_i) \cap \mathcal{B}_0$ **do**
7   s_i transmits the broadcast data packet to s_j by unicast when there is a spectrum opportunity for this transmission;
8                  $\mathcal{B}_1 \leftarrow \mathcal{B}_1 \cup \{s_j\}$;
9                  $\mathcal{B}_0 \leftarrow \mathcal{B}_0 \setminus \{s_j\}$;

10 Tessellate the CRN with half-open half closed hexagons of radius $\frac{r}{2}$ and give a $3k^2$-coloring to the tessellation, where $k = 2$;
11 Partition the dominators $\mathcal{D}$ into $3k^2$ disjoint subsets $\mathcal{D}_1, \mathcal{D}_2 \cdots, \mathcal{D}_{3k^2}$ according to the method discussed in Section IV;
12 $\mathcal{B}_0 \leftarrow V_s \setminus \mathcal{B}_1$;
13 **while** $\mathcal{B}_0 \neq \emptyset$ **do**
14   Schedule $\mathcal{D}_1, \mathcal{D}_2 \cdots, \mathcal{D}_{3k^2}$ repeatedly each for one time slot by calling procedure BROAD-UNI-CAST($\mathcal{D}_i$);

Now, we are ready to present the broadcast scheduling algorithm. Based on the constructed broadcasting tree in Section IV, we propose a Mixed Broadcast Scheduling (MBS) algorithm under the UDG model, named MBS-UDG, as shown in Algorithm 1. From Algorithm 1, we can see that both the unicast communication mode and the broadcast communication mode are exploited to finish the broadcast scheduling. Basically, Algorithm 1 consists of two phases: Phase I (lines 1-6) is used to broadcast the data packet to all the dominators and connectors of T, i.e. $\{s_0\} \rightarrow \mathcal{D} \cup \mathcal{C}$; and Phase II (lines 7-10) is used to broadcast the data packet to all the dominatees of T, i.e. $\mathcal{D} \rightarrow V_s \setminus (\mathcal{D} \cup \mathcal{C})$. In Phase I, the broadcast scheduling is finished by unicast. In Phase II, we first tessellate the network plane with hexagons. Subsequently, we partition the dominator set $\mathcal{D}$ into disjoint subsets by

[2]The situation that two SUs choose the same sensing points can be tackled by assigning them different sensing points in a centralized manner instead of randomly choosing, or by an exponential backoff mechanism

coloring the tessellation. Finally, since every dominatee has a dominator parent in T, these dominator subsets are scheduled repeatedly until all the dominatees received the data packet. When schedule each dominator subset, the procedure BROAD-UNI-CAST($\mathcal{D}_i$) (Algorithm 2) is called. From Algorithm 2, we can see that BROAD-UNI-CAST($\mathcal{D}_i$) finishes the broadcast scheduling by employing mixed unicast and broadcast. For a SU $s_i \in \mathcal{D}_i$, if the number of its children that have not received the data packet is no less than a threshold $\exp(\pi\lambda(r^2+R^2))$, it will broadcast the data packet. Otherwise, it will transmit data to the children that wait for receiving the broadcast packet by unicast.

Algorithm 2: BROAD-UNI-CAST($\mathcal{D}_i$)

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1 **if** $\forall s_u \in \mathcal{D}_i, \ c(s_u) \cap \mathcal{B}_0 = \emptyset$ **then**
2 $\quad$ **return**
3 $\forall s_u \in \mathcal{D}_i$, **if** $|c(s_u) \cap \mathcal{B}_0| > \exp(\pi\lambda(r_I^2 + R_I^2))$ **then**
4 $\quad$ s_u broadcasts the data packet to its children in $c(s_u) \cap \mathcal{B}_0$ when its transmission does not cause any unacceptable interference to the primary network;
5 **else if** $0 < |c(s_u) \cap \mathcal{B}_0| \leq \exp(\pi\lambda(r_I^2 + R_I^2))$ **then**
6 $\quad$ For every $s_v \in (c(s_u) \cap \mathcal{B}_0)$, s_u transmits the data packet to it by unicast when there is a spectrum opportunity for this transmission;
7 $\forall s_v \in (c(s_u) \cap \mathcal{B}_0)$, **if** s_v *has successfully received the broadcast data packet from* s_u **then**
8 $\quad$ $\mathcal{B}_0 \leftarrow \mathcal{B}_0 \setminus \{s_v\}$;

In MBS-UDG, we employ unicast sometimes instead of broadcast. This is mainly because the spectrum opportunities of the neighbors of a SU vary over time. Consequently, a broadcast operation not necessarily has more effective receivers than a unicast operation. In some cases, a broadcast may not have any effective receiver due to lack of spectrum opportunities, which is common in CRNs. We further take an example to analyze the reasons. For a SU s_i carrying the broadcast packet, suppose it has three children s_1, s_2, and s_3 waiting for receiving the data from s_i. Assume s_1, s_2, and s_3 have spectrum opportunities to receive the data at the 5-th time slot, at the 10-th time slot, and at the 15-th time slot, respectively. Then, if s_i keeps broadcasting the data packet, 15 time slots are needed to let its all children to receive the data, and s_i broadcasts the data packet for 15 times. On the other hand, if s_i exploits unicast to finish this broadcast task, s_i only has to transmit the packet for 3 times without increasing the latency. Evidently, in these situations, unicast has advantages, e.g. less data transmission times, less energy consumption, less interference to other ongoing communications, and etc., over broadcast. Therefore, by carefully designing, we properly employ unicast or broadcast based on different situations in MBS-UDG. This can also be seen from the following lemma, which indicates how many dominatee children can successfully receive the broadcast packet from s_u during one

transmission (broadcast or unicast) of s_u. The proof of Lemma 3 is omitted due to space limitation.

Lemma 3: In Algorithm 2, the expected number of dominatee children that can successfully receive the broadcast packet from each data transmission of $s_u \in \mathcal{D}_i$ is at least 1.

B. Analysis of MBS-UDG

In this subsection, we analyze the latency and redundancy performance of MBS-UDG.

1) Broadcast Latency of MBS-UDG: Since MBS-UDG consists of two phases, we will analyze the latency of these two phases respectively. First, based on the defined network model, we can obtain the spectrum opportunity for a pair of SUs as shown in Lemma 4. The proof of Lemma 4 is omitted due to space limitation.

Lemma 4: Suppose $(s_u, s_v) \in E_s$ in $G = (V_s, E_s)$. The spectrum opportunity for s_u to transmit a data packet to s_v is $p = \exp(-\pi\lambda(r_I^2 + R_I^2))$.

Since $r_I = r$ and $R_I = R$ under the UDG model, let $p = \exp(-\pi\lambda(r^2 + R^2))$ in the following of this section. In Phase I, the task is to broadcast the data packet to all the other dominators and connectors of T from the data source (root) s_0. According to the construction process of T, we know that for $\forall s_v \in \mathcal{C}$, its parent node in T is a dominator, and $\forall s_v \in \mathcal{D}$, if $s_v \neq s_0$, its parent node in T is a connector. For convenient, without specification, we use (s_u, s_v) to represent the communication that s_u transmits the broadcast data packet to s_v in the following of this paper. Then, we have the following lemma, which indicates the time consumption to transmit the broadcast packet from a parent dominator to a child connector. The proof of Lemma 5 is omitted due to space limitation.

Lemma 5: Suppose $s_v \in \mathcal{C}$ is waiting for receiving the broadcast packet, and $s_u = p(s_v) \in \mathcal{D}$ is currently holding the broadcast packet. Then, the expected time for s_v to receive the broadcast packet from s_u in MBS-UDG is at most $262\tau/p$.

By similar technique, we can obtain the time consumption to transmit the broadcast packet from a connector parent to a dominator child as shown in Lemma 6. The proof of Lemma 6 is omitted due to space limitation.

Lemma 6: Suppose $s_v \in \mathcal{D}$ is waiting for receiving the broadcast packet, and $s_u = p(s_v) \in \mathcal{C}$ is currently holding the broadcast packet. Then, the expected time for s_v to receive the broadcast packet from s_u in MBS-UDG is at most $472\tau/p$.

Based on Lemma 5 and Lemma 6, we can obtain the time consumption of Phase I as shown in Theorem 1.

Theorem 1: In MBS-UDG, the expected time consumption of Phase I is at most $367(\hbar - 1)\tau/p$.

Proof: Suppose s_v is the last SU to receive the broadcast packet in Phase I of MBS-UDG. Then, s_v must be a dominator. This is because each connector must have a dominator parent and at least one dominator child. Otherwise, that connector will not be selected to the minimal connector set based on the construction process of T. Now, suppose the data transmission path from s_0 to s_v is $\mathcal{P} : s_0 \to s_a \to \cdots \to s_u \to s_v$. Then,

the transmissions on $\mathcal{P}$ are *dominator→connector communications* and *connector→dominator communications* alternately. Therefore, based on Lemma 5 and Lemma 6, the expected time consumption for s_v to receive the broadcast packet is at most $\frac{h(s_v)}{2} \cdot (262\tau/p + 472\tau/p) = 367 h(s_v)\tau/p$, where $h(s_v)$ is the height of s_v in T. Furthermore, $h(s_v) \leq \hbar - 1$, where $\hbar$ is the height of T. It follows this theorem holds. $\qquad\square$

Now, we analyze the time consumption of Phase II in MBS-UDG. In Phase II, only dominators and dominatees are involved in the broadcasting process. We partition dominators into $3k^2 = 3 \times 2^2 = 12$ disjoint subsets $\mathcal{D}_i$ ($1 \leq i \leq 12$), and schedule these subsets repeatedly by calling procedure BROAD-UNI-CAST($\mathcal{D}_i$) until all the dominatees received the broadcast packet.

To obtain the time consumption of Phase II, we first analyze the time consumption for $\mathcal{D}_i$ to transmit the broadcast packet to all the SUs in $\bigcup_{s_u \in \mathcal{D}_i} c(s_u) \setminus \mathcal{C}$. From Lemma 2 and the discussion in Section IV-B, we know that for $\forall s_u, s_v \in \mathcal{D}_i$ ($1 \leq i \leq 12$), $D(s_u, s_v) > \frac{(3 \times 2 - 2)r}{2} = 2r$, which implies all the SUs in $\mathcal{D}_i$ can transmit data simultaneously under the UDG model as long as they have spectrum opportunities. Then, we have the following Lemma 7 which indicates the time consumption of procedure BROAD-UNI-CAST($\mathcal{D}_i$). The proof of Lemma 7 is omitted due to space limitation.

Lemma 7: The expected number of time slots consumed by dominators in $\mathcal{D}_i$ to transmit the broadcast packet to all the dominatees in $\bigcup_{s_u \in \mathcal{D}_i} c(s_u) \setminus \mathcal{C}$ by BROAD-UNI-CAST($\mathcal{D}_i$) is at most Δ_T/p if $\Delta_T \leq 1/p$, and $1/p^2 + \exp(\pi\lambda r^2)\log_{1-p}\frac{1}{p\Delta_T}$ if $\Delta_T > 1/p$.

Based on Lemma 7, we can obtain the time consumption of Phase II of MBS-UDG as shown in Theorem 2.

Theorem 2: The expected number of time slots consumed by Phase II of MBS-UDG is at most $12\Delta_T/p$ if $\Delta_T \leq 1/p$, and $12/p^2 + 12\exp(\pi\lambda r^2)\log_{1-p}\frac{1}{p\Delta_T}$ if $\Delta_T > 1/p$.

Proof: Since $\mathcal{D}$ can be partitioned into 12 disjoint and interference-free subsets, this theorem can be obtained based on the conclusion of Lemma 7. $\qquad\square$

MBS-UDG consists of Phase I and Phase II, and these two phases are executed sequentially. Consequently, the time consumption of MBS-UDG can be obtained directly from Theorem 1 and Theorem 2 as follows.

Theorem 3: The expected time consumption of MBS-UDG is upper bound by $(367(\hbar-1) + 12\Delta_T)\tau/p$ if $\Delta_T \leq 1/p$, and $(367(\hbar-1)/p + 12/p^2 + 12e^{\pi\lambda r^2}\log_{1-p}\frac{1}{p\Delta_T})\tau$ if $\Delta_T > 1/p$.

2) Broadcast Redundancy of MBS-UDG: Now, we analyze the broadcast redundancy of MBS-UDG, which is defined as the maximum transmission times of the broadcast packet by any SU during the entire broadcast scheduling (see Section III). According to Algorithm 1, we know that for any dominatee, its broadcast redundancy is 0 since it does not transmit the broadcast packet. For any $s_u \in \mathcal{C}$, it transmits the broadcast packet to its children, which are dominators (see the construction process of T), by unicast according to Algorithm 1. Therefore, the broadcast redundancy for $\forall s_u \in \mathcal{C}$ is at most 4 since it has at most 4 dominator children based on Lemma 1. For any $s_u \in \mathcal{D}$, it has at most 12 connector children which

have to receive the broadcast packet by unicast according to Algorithm 1. For its dominatee children, it will transmit at most $\Delta(s_u) \leq \Delta_T$ times by unicast if $\Delta(s_u) \leq 1/p$ according to Algorithm 2. Otherwise, it will broadcast for $\log_{1-p}\frac{1}{p\Delta(s_u)} \leq \log_{1-p}\frac{1}{p\Delta_T}$ times and unicast for $1/p$ times. Therefore, for $s_u \in \mathcal{D}$, its broadcast redundancy is at most $12 + \Delta_T$ when $\Delta(s_u) \leq 1/p$, and $12 + 1/p + \log_{1-p}\frac{1}{p\Delta_T}$ when $\Delta(s_u) > 1/p$. In summary, we use the following theorem to show the broadcast redundancy of MBS-UDG.

Theorem 4: The broadcast redundancy of MBS-UDG is at most $12 + \Delta_T$ when $\Delta_T \leq 1/p$, and $12 + 1/p + \log_{1-p}\frac{1}{p\Delta_T}$ when $\Delta_T > 1/p$.

C. Broadcast Scheduling under the PrIM

Under the UDG model, the interference range of a SU/PU is assumed to be same as its transmission range. In reality, the interference range of a node is usually larger than its transmission range. Therefore, a more general interference model for CRNs is the *PrIM*, where the interference range of a SU/PU is β times of its transmission range as defined in Section III, i.e. $r_I = \beta r$ and $R_I = \beta R$, where $\beta \geq 1$ is a constant. Based on Lemma 4, let $p_\beta = \exp(-\pi\lambda\beta^2(r^2 + R^2))$ to denote the unicast opportunity under the PrIM in this section.

To make proposed MBS algorithm work under the PrIM, we need to change the k in MBS-UDG to $k = \lceil\frac{2\beta+4}{3}\rceil$. Then, according to Lemma 2, for $\forall s_u, s_v \in \mathcal{D}_i$, $D(s_u, s_v) > \frac{(3k-2)r}{2} \geq (\beta+1)r$, which implies all the SUs in $\mathcal{D}_i$ can conduct data transmission simultaneously as long as they have spectrum opportunities. The new MBS algorithm under the PrIM is denoted by MBS-PrIM. By similar techniques of analyzing the latency and redundancy of MBS-UDG, we can obtain the broadcast latency and redundancy of MBS-PrIM as follows: (i) let $k = \lceil\frac{2\beta+4}{3}\rceil$. The expected number of time slots consumed by MBS-PrIM is at most $((6\phi_{\beta+1} + 6\phi_{\beta+2} - 11)(\hbar-1) + 3k^2\Delta_T)/p_\beta$ if $\Delta_T \leq 1/p_\beta$, and $(6\phi_{\beta+1} + 6\phi_{\beta+2} - 11)(\hbar-1)/p_\beta + 3k^2/p_\beta^2 + 3k^2\exp(\pi\lambda\beta^2 r^2)\log_{1-p_\beta}\frac{1}{p_\beta\Delta_T}$ if $\Delta_T > 1/p_\beta$; and (ii) the broadcast redundancy of MBS-PrIM is at most $12 + \Delta_T$ when $\Delta_T \leq 1/p_\beta$, and $12 + 1/p_\beta + \log_{1-p_\beta}\frac{1}{p_\beta\Delta_T}$ when $\Delta_T > 1/p_\beta$.

VI. SIMULATION AND ANALYSIS

In this section, we examine the latency and redundancy performance of MBS by simulations. In all the simulations, we assume a secondary network coexisted with a primary network distributed in a square area of size $X \times Y$, and they share a same common channel (detailed network model can be found in Section III). The primary network is Poisson distributed and the activity of PUs satisfies a two-dimensional Poisson point process with density λ (see Section III). In the secondary network, a broadcast source SU which carries a broadcast packet and n other SUs are randomly distributed. The *network density*, denoted by ρ, of the secondary network is defined as the ratio between n and the network size. The network time is slotted with each time slot of length 1 millisecond (ms). For
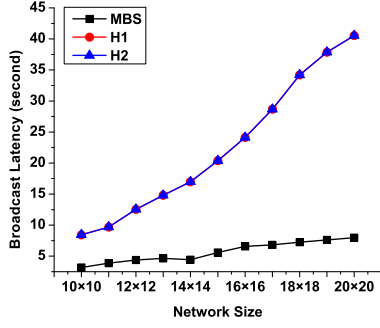
(a)
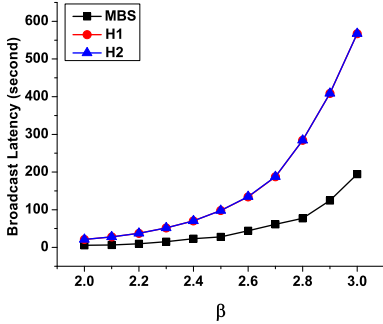
(b)

Fig. 3. Broadcast latency of MBS. Without specifications, the default settings are $X = 15, Y = 15, \rho = 4.0, \lambda = 0.15, r = 1.0, R = 1.0$, and $\beta = 2.0$.

other network settings, e.g. r, R, β and etc defined in Section III, we will specify them in each group of simulations.

Since the UDG model is a special case of the PrIM, we consider the PrIM in all the simulations. Furthermore, we use MBS to denote our algorithm. The compared algorithms are the two heuristic algorithms proposed in [12], denoted by H1 and H2 respectively, which are the only algorithms for the MLBS problem in CRNs. In both H1 and H2, only the broadcast communication mode is employed. The basic idea of H1 is that each SU broadcasts the data packet to its child which has the path to the farthest SU (from the source SU) first. On the other hand, H2 tries to maximize the number of children that can receive the broadcast packet during each transmission at each step. In the following, we will compare MBS with H1 and H2 for both broadcast latency and redundancy. Each group of simulations is repeated for 100 times, and the results are the average values of these 100 times.

A. Broadcast Latency of MBS

In this subsection, we check the latency performance of MBS, H1, and H2 under different scenarios. If we fix the density of SUs as $\rho = 4.0$, the impacts of the network size on the broadcast latency of MBS, H1, and H2 are shown in Fig.3 (a). From Fig.3 (a), we can see that the induced latency of all the three algorithms increases when the network size increases. The reason is straightforward since more SUs are involved

in the broadcast task and the height of the broadcasting tree increases. MBS has better performance than H1 and H2, especially in large CRNs. This is because a successful broadcast opportunity (the opportunity that all the neighbors of a SU can receive the broadcast packet from that SU by one broadcast communication) is usually much rarer than a successful unicast opportunity. By taking this advantage, MBS first transmits the broadcast packet to the selected backbone SUs by unicast. Subsequently, MBS schedules these backbone SUs concurrently by employing mixed unicast and broadcast communication modes to achieve maximum transmission efficiency (see the analysis in Section V-A), which significantly accelerates the broadcast scheduling. Furthermore, H1 and H2 have similar performance, which is consistent with the results in [12]. On average, MBS induces 298.04% and 298.16% less time compared with H1 and H2 respectively.

When increase β (the coefficient in the PrIM), the changes of the latency of the three algorithms are shown in Fig.3 (b). From Fig.3 (b), we can see that when β increase, the induced latency of MBS, H1, and H2 increases. This is because a large β implies large interference area of both SUs and PUs. Hence, both the transmission concurrency and the spectrum opportunities for all the algorithms decrease, and followed by more time consumption. Again, MBS takes less time than H1 and H2 to finish the broadcast scheduling since it exploits mixed unicast and broadcast communication modes depending on different network environments.

B. Broadcast Redundancy of MBS

We examine the broadcast redundancy performance of MBS, H1, and H2 in this subsection and the results are shown in Fig.4. Fig.4 (a) shows the impacts of network size on the redundancy of MBS, H1, and H2. From Fig.4 (a), we can see that if we fix the network density and change the network size, the broadcast redundancy of all the three algorithms keeps stable. This is because fixed network density implies the number of children of an inner node in the broadcasting tree keep stable no matter the size of the network/tree. On the other hand, the broadcast redundancy depends on the number of children that a node have and the available spectrum opportunities. Therefore, network size has a little impact on the redundancy of all the three algorithms. Furthermore, MBS has a better performance than H1 and H2. This is because H1 and H2 employ the broadcast communication mode only. As we analyzed in Section V-A, a broadcast transmission in CRNs may not have any receiver due to the lack of spectrum opportunities. Therefore, a SU in H1 and H2 may broadcast many times to guarantee its children to receive the broadcast packet, which also wastes the energy of a SU and cause more interference to both the primary network and the secondary network. In MBS, the unicast and broadcast communication modes are all employed depending on network scenarios to maximize the transmission efficiency. Consequently, MBS is more efficient on broadcast redundancy. On average, MBS induces 63.61% and 63.84% less redundancy than H1 and H2, respectively.
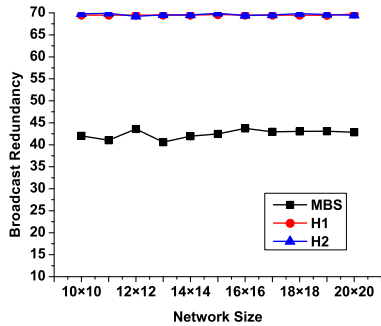
(a)
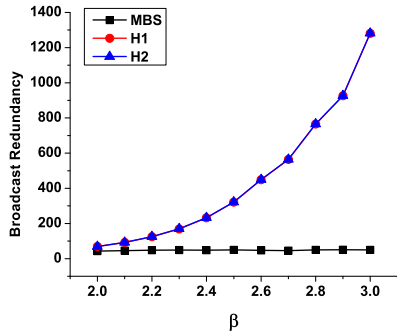
(b)

Fig. 4. Broadcast redundancy of MBS. Without specifications, the default settings are $X = 15, Y = 15, \rho = 4.0, \lambda = 0.15, r = 1.0, R = 1.0,$ and $\beta = 2.0$.

The impacts of β on the redundancy of MBS, H1, and H2 are shown in Fig.4 (b). From Fig.4 (b), we can see that when β increases, the redundancy of H1 and H2 increases significantly. This is because their redundancy highly depends on the broadcast spectrum opportunity, which is further affected by β. A large β implies large interference area for both SUs and PUs. Therefore, large β induces less spectrum opportunities and followed by high redundancy in H1 and H2. On the other hand, the dominating factor to decide the redundancy of MBS is the number of children a backbone SU has. Therefore, a large β has significant impacts on the latency of MBS, however small impacts on the redundancy of MBS. Consequently, the redundancy of MBS keeps almost stable. On average, H1 and H2 induce 8.4637 and 8.467 times of more redundancy than that of MBS, respectively.

VII. Conclusion

Broadcast is one of the most fundamental operations in wireless networks, as well as in CRNs. However, on one hand, it is not trivial to extend traditional broadcasting methods to CRNs. On the other hand, existing broadcasting algorithms for CRNs are either heuristic solutions without performance guarantee or with performance far from the optimal solution. Therefore, we study the MLBS problem for CRNs in this paper. First, we propose a Mixed Broadcast Scheduling (MBS) algorithm under the UDG model, denoted by MBS-UDG. We obtain the latency performance of MBS-UDG, which significantly improves existing results. Furthermore, the redundancy performance of MBS-UDG is also obtained. Second, we generalize MBS to CRNs under the PrIM, and analyze the algorithm latency and redundancy performance under that model. Finally, simulations are conducted to examine the performance of MBS, which demonstrate that MBS significantly improves both the broadcast latency and the broadcast redundancy over existing algorithms.

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